Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1		In a binomial distribution, if the mean is 6, and the variance is 4, find P[X=1].	3
2		Given that f(x) = $\frac{K}{2^x}$ is a probability mass function of a random variable that can take on the values x = 0,1,2,3 and 4, find (i) K and (ii) $P(X \leq 2)$.	3
3		Find the mean and variance for the PDF, f(x) = $\begin{cases} Kx^2 & ,0 < X < 1 \\ 0 & ,elsewhere \end{cases}$	3
4		If random variable X has a uniform distribution in (-3,3), find P ($\mid X - 2 \mid < 2$).	3
5		Define stationary random process. Define two types of stationary random process.	3
6		Write down the properties of the power spectral density.	3
7		Write down the Newton's forward and backward difference interpolation formula	3
8		Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with 4 subintervals by Simpson's rule.	3
9		Write the normal equations for fitting a curve of the form y = a + bx + cx^2 to a given set of pairs of data points.	3
10		Using Euler's method, find y at x = 0.25, given y^{I} = 2xy, y (0) =1, h= 0.25.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	Six dice are thrown 729 times. How many times do you expect at least three dice to show 1 or 2?	6
	b)	Derive the formula for mean and variance of Poisson distribution	8
12	a)	A random variable X takes the values -3, -2, -1,0,1,2,3 such that P(X=0) = P(X>0) = P(X<0) and P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3). Obtain the probability mass function and distribution function of X.	7

	b)	The joint probability distribution of X and Y is given by, f(x ,y) = $\frac{1}{27}(2x+y)$; x =0,1,2 and y = 0,1,2. (i) Find the marginal distributions of X and Y. (ii) Are X and Y independent random variables.	7

Module -2

13	a)	Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8.(i) What is the probability that the diameter of a randomly selected tree will be at least 10 in.? (ii) What is the probability that the diameter of a randomly selected tree will exceed 20 in.? (iii) What is the probability that the diameter of a randomly selected tree will be between 5 in. and 10 in.?	7
	b)	The amount of time that a surveillance camera will run without having to be reset is a random variable having exponential distribution with mean 50 days. Find the probabilities that such a camera will (a) have to be reset in less than 20 days. (b) not have to be reset in at least 60 days.	7
14	a)	The joint density function of 2 continuous random variable X and Y is f (x,y)=$\begin{cases} cxy \; ; & 0 < x < 4 \,,\; 1 < y < 5 \\ 0 \; ; & otherwise \end{cases}$ (i) Find the value of the constant c. (ii) Find P (X ≥ 3, Y ≤ 2) (iii) Find the marginal density of X.	7
	b)	The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem, find the probability that the average life time of 60 lights exceeds 1250.	7

Module -3

15	a)	Let X(t) = A Cos λt + B Sin λt, where A and B are independent normally distributed random variables N (0,σ^2). Show that X(t) is WSS.	7
	b)	If X(t) = A Cos $(\omega t+\theta)$ Where A and ω are constants and θ is uniformly distributed over [0,2π], find the auto correlation function and Power Spectral Density of the process.	7

16	a)	Assume that X(t) is a random process defined as follows: X(t) = A cos (2πt + ∅) where A is a zero-mean normal random variable with variance σ_A^2 = 2 and ∅ is uniformly distributed random variable over the interval −π ≤ φ ≤ π. A and φ are statistically independent. Let the random variable Y be defined as Y = $\int_0^1 X(t)dt$. Determine (i) The mean of Y (ii) The variance of Y.	7
	b)	If the customers arrive at a counter in accordance with Poisson distribution with rate of 2 per minute. Find the probability that the interval between two consecutive arrivals is (i) more than 1 minute (ii) between 1 minute and 2 minutes.	7

Module -4

17	a)	Use Newton- Raphson method to find a non- zero solution of f(x)=2x - Cos x = 0	7
	b)	Using Lagrange's interpolating polynomial estimate y (5) for the following data:	7

x	1	3	4	6
y	-3	0	30	132

18	a)	Find the polynomial interpolating the following data, using Newton's backward interpolating formula	7

x	3	4	5	6	7
y	7	11	16	22	29

	b)	Using Newton's divided difference formula, evaluate y(8) and y(15) from the following data	7

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

Module -5

19	a)	Solve the following system of equations using Gauss- Seidel iteration method starting with the initial approximation $(0,0,0)^T$	7

$8x_1 + x_2 + x_3 = 8$

$2x_1 + 4x_2 + x_3 = 4$

$x_1 + 3x_2 + 5x_3 = 5$

	b)	Fit a straight-line y = ax + b for the following data:	7

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

20 a) Solve the following system of equations using Gauss- Jacobi iteration method starting with the initial approximation $(0,0,0)^T$ 7

$$20x_1 + x_2 - 2x_3 = 17$$

$$3x_1 + 20x_2 - x_3 = -18$$

$$2x_1 - 3x_2 + 20x_3 = 25$$

b) Use Runge - Kutta method of fourth order to find y (0.1) from $\frac{dy}{dx} = \sqrt{x+y}$, y (0) =1 taking h = 0.1. 7

***